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Dictatorship from majority rule voting

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Abstract. Majority rule voting in a multi-level system is studied using tools from the physics of disorder. We are not dealing with nation-wide general elections but rather with hierarchical organisations made of small committees. While in theory, for a two candidate election, the critical threshold to absolute power is 50%, the usual existence of some local and reasonable bias makes it asymmetric, transforming a democratic system in effect to a dictatorship. The underlying dynamics of this democratic self-elimination is studied using a simulation which visualizes the full process. In addition the effect of non-voting persons (abstention, sickness, apathy) is also studied. It is found to have an additional drastic effect on the asymmetry of the threshold value to power. Some possible applications are mentioned.

PACS. 05.45-a Nonlinear dynamics and nonlinear dynamical systems – 87.23.Ge Dynamics of social systems – 87.23.Kg Dynamics of evolution – 05.50+q Lattice theory and statistics (Ising, Potts, etc.)

In this paper we are going from physics to politics. While in recent years statistical physics has been applied to a large spectrum of fields far outside the scope of non-living matter, its application to social sciences is still very scarce [1].

Here we use some concepts from physics of disorder [2] to analyse a basic feature of social institutions, namely the process of leadership election within hierarchical organizations. We are not dealing here with nation-wide general elections. Instead we are studying hierarchical organisations made of small committees.

In democratic hierarchies, each level is determined from the one just below using a local majority rule. In principle the rule is 100% power to the larger group which in case of two candidates, should score more than 50% support from the total population.

Studying the underlying dynamics associated with such multi-level elections, it is found to lead to a phenomenon of huge majority democratic self-elimination against a minority in power [3]. Indeed, repeated elections can drive the threshold to power from 50% to some asymmetric value. For instance, it can simultaneously be down to 23% for one group while still in a winning position, and up to 77% for its competitor, which is still losing.

These effects provide a new explanation to the observed difficulty in overthrowing a ruling group. In other words, we are determining the democratic conditions under which a small minority keeps in power for very long times, against the *a priori* democratic criterium of 50%. Moreover, the existence of non-voting people, which is a growing major feature in recent western country elections, is demonstrated to drastically shift the threshold away from 50%.

At this stage it is worth to state that we aim to emphasize a trend associated with majority rule voting rather than to explain all details of democratic elections. Moreover we are focusing on small size committees. Indeed a similar process was shown to make reforms difficult [4]. A diffusive version was also studied in the context of emergence of new species [5,6].

We now present our results using the simplest version of the model. Let us start from local cells constituted by a small number of individuals. Two politics, Green (G) and Red (R), are available, the first one is supported by p_0 of the whole population while $(1-p_0)$ support the second one. We are also assuming each person does have an opinion. First, cells of some size are formed randomly (*e.g.* home localization or working place) from the overall population. Then each cell elects a representative, either a Red or a Green using a local majority rule. These elected people constitute the first hierarchical level of the organization called level 1. New cells are then formed at level 1 from these elected people. They in turn elect new representatives to build level 2. This process is repeated again and again until we reach a single person at the top of the hierarchy (see Fig. 1).

For the sake of demonstration we take the case of even cells, and in particular cells of four persons. We are interested in having the possibility of ties in order to account for the bias always existing somehow in favor of the ruling party. In most social situations it is well admitted and

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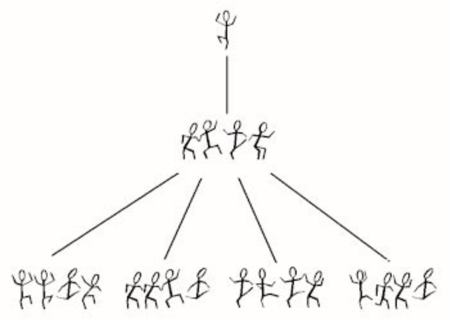

Fig. 1. The person at the top of the hierarchy is elected by the members of the middle level. These persons are in turn elected by small groups from the main population.

even well understood that to change a policy requires a majority. Therefore, in case of a tie, things stay as they are. This is a bias in favor of the ruling leadership. It is often achieved in a more subtle way, for instance by giving one additional vote to the committee president. Within our model, it means that a 2R–2G tie cell votes for a Red assuming the Red was in power first.

For this reason the associated voting function becomes asymmetrical. The probability for a Green to be elected at level $n+1$ by people at level n, is

$$p_{n+1} = p_n^4 + 4p_n^3(1 - p_n), \quad (1)$$

where p_n is the portion of elected Green persons at level n. In contrast for a Red to be elected the probability is

$$1 - p_{n+1} = (1 - p_n)^4 + 4(p_n - 1)^3(1 - p_n) + 2p_n^2(1 - p_n)^2, \quad (2)$$

where last term embodies the bias in favor of the Red in case of a tie. The Green-voting function has two stable fixed points: 0 and 1. In between an unstable fixed point determines the threshold to full power (reached at $p_l = 1$) when above, or to total disappearance (reached at $p_m = 0$) when below. From equation (1) it is

$$p_{c,4} = \frac{1 + \sqrt{13}}{6} = 0.7676, \quad (3)$$

which makes the threshold to power for Green at about 77%, far above the supposed value of 50% (see Fig. 2).

Moreover the process of democratic self-elimination of an initial huge majority is rather quick when climbing up the hierarchy. Only a small number of levels is needed to reduce the Green representation to zero. For instance, starting from $p_0 = 0.63$ only 5 levels are needed to lower the Green fraction to less than a percent. The associated series is $p_1 = 0.53$, $p_2 = 0.36$, $p_3 = 0.14$ and $p_4 = 0.01$.

A reasonable bias in favor of the Reds has thus turned a majority rule democratic voting system to a totalitarian representation. To get to power Green must pass over 77% of support which is nearly out of reach in any democratic country.

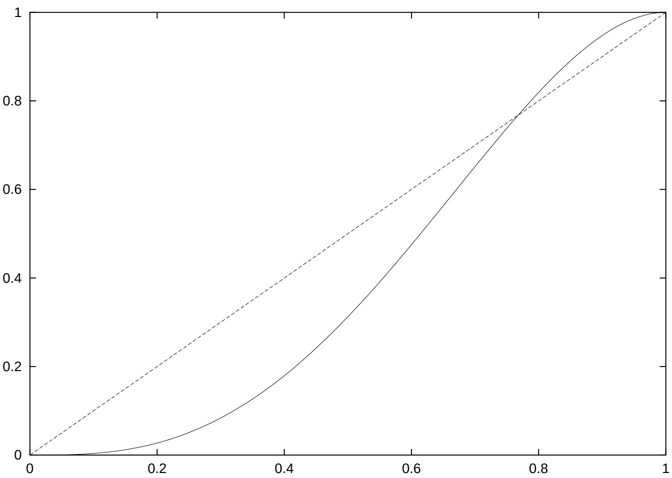

Fig. 2. The solid black line is the Green voting function given by equation (1). The dashed straight line just helps visualizing the unstable fixed point at 77%.

To illustrate the dynamics of our model more dramatically, we now present a series of visualizations. The four pictures are all from a single simulation run that started with a situation where the Reds were in total power (somewhat like in Russia immediately after the revolution). Then the support for Green slowly starts growing, and the visualization shows what is happening.

Mechanically, the simulation is done the following way: A single voter is chosen randomly and is "asked" for his opinion, simply by generating a random number and comparing this to p_0. If the random number is lower than p_0, the voter decides for Green, otherwise he will stay Red. Subsequently, a count is taken of his subcell, and a new representative is chosen if needed. Ties are always broken in favor of Red. If the representative has changed, the representative's subcell is counted, and a new, higher level representative is elected. This process repeats, climbing up the ladder of the hierarchy until the president is reached. Note that only on the ground level of the hierarchy the voters have any freedom to decide, the upper level representatives are bound by the opinion of their supporters.

Figure 3a shows a situation where 55% of the main population is in support of the Green party, thus over the democratic threshold to take over. However, the hierarchical voting process eliminates this majority after only three additional levels. Surprisingly, the situation is not much improved when the ground support for Green reaches up to 70%, as illustrated by Figure 3b. There still is next to no support for the Green party two levels below the president. Even at 76.4% Green support, just below the threshold value from equation (3), the president still is Red against a huge majority of more than three quarters. Only once the Green support goes above the threshold value as shown in Figure 3d, the Red government is overthrown and the Greens move into power.

At this stage, though very strong, our results depend essentially on the existence of the tie bias which could be argued to be purely academic. Moreover we must note that

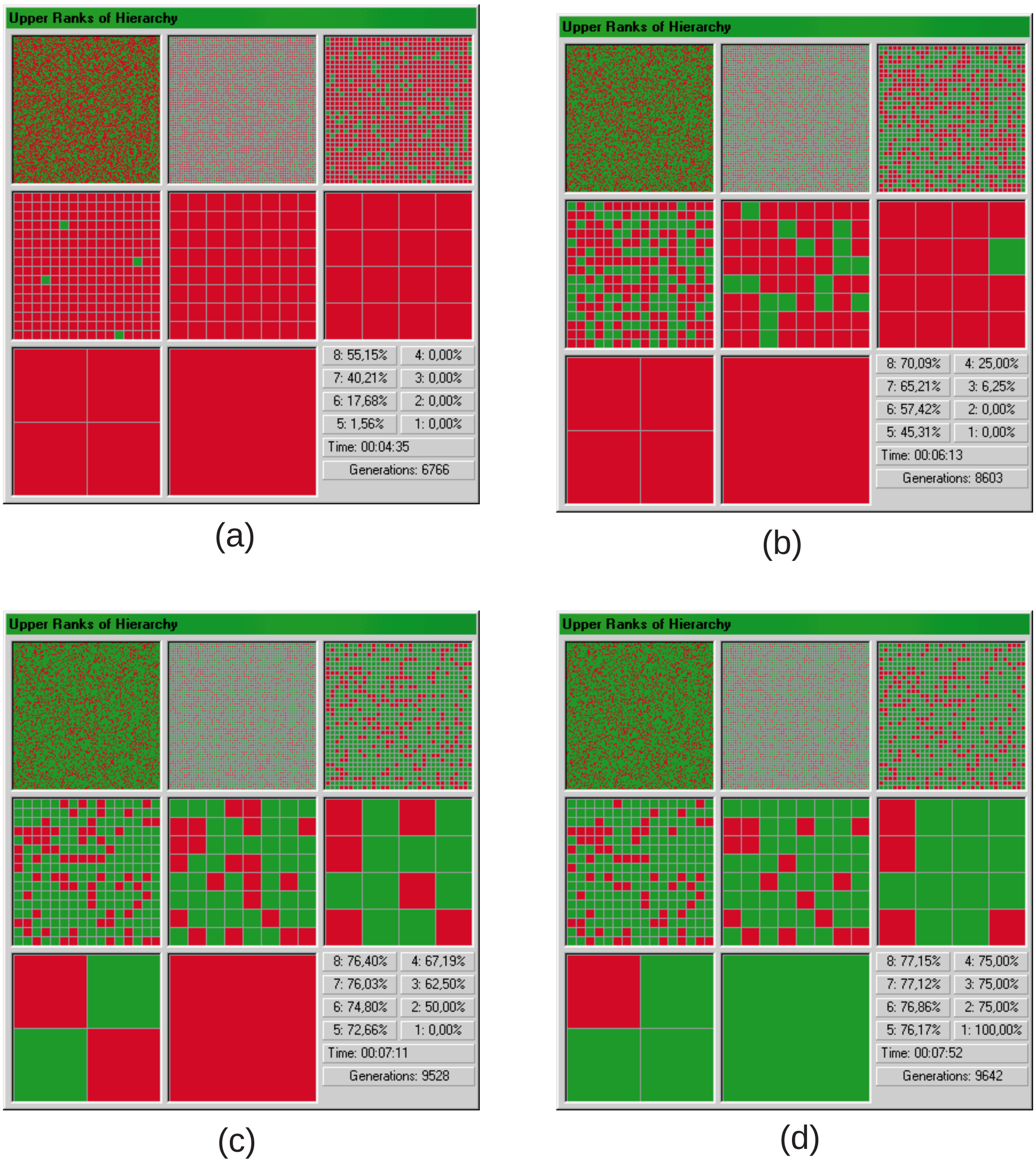

Fig. 3. Four situations of a simulation with eight levels of hierarchy. (a) at 55% green fraction, the majority is eliminated after two additional levels. (b) at 70% the situation is not much improved (c) Red still clinging to power just below the critical threshold (d) Green has taken power.

the threshold value from equation (3) decreases asymptotically from above towards 50% with increasing even sizes. However, we will demonstrate that such biases are indeed very general and resist increasing sizes. To do so, we now introduce in the model the contemporary quite well spread phenomenon of non-voting.

To make our argument stronger, we will start from the case of a three-cell model, which has its threshold at 50% when there is no abstention. We then assume that each person has a probability q of voting and $(1-q)$ of non-voting by will, sickness, apathy, or any other reason. Consequently we have to determine the elective scheme in cases the cell is not full, *i.e.*, what happens for two, one or zero voting persons. Indeed this non-voting effect *de facto* introduces a bias similar to the one we had before.

The same principle "a majority is required to change things" gives for both a non-voting cell (3 non-voting persons) and a tied 1R–1G cell (one non-voting person) an elected Red representative. Therefore the probability to have a Green elected at level $n+1$ by people at level n becomes

$$p_{n+1} = q^3(p_n^3 + 3p_n^2(1-p_n)) + 3q^2(1-q)p_n^2 + 3q(1-q)^2 p_n, \quad (4)$$

where the last term accounts for a one voting person case. For $q = 1$ (everyone voting) this voting function has $p = 0$

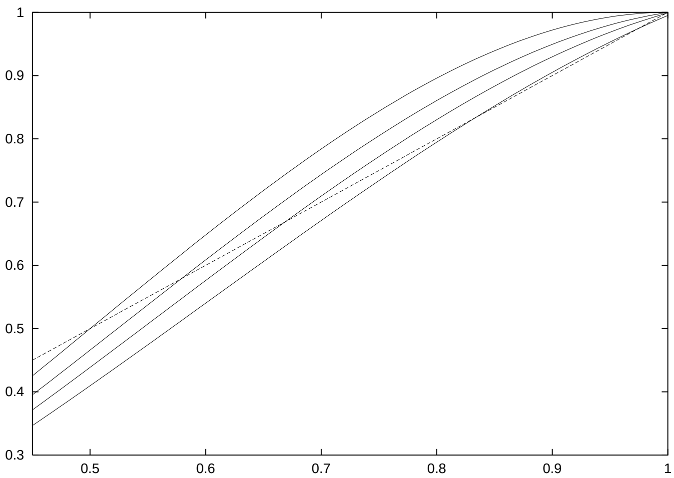

Fig. 4. Voting function given by equation (4) for $q = 1.00$, $q = 0.95$, $q = 0.90$ and $q = 0.83$, in descending order. The dashed line helps visualizing the movement of the two fixed points towards each other with descending q.

and $p = 1$ as stable fixed points and $p = \frac{1}{2}$ as the unstable critical threshold.

From equation (4), it is seen as soon as $q \neq 1$, the two fixed points at $p = 1$ and $p = \frac{1}{2}$ move towards one another to merge at $q \sim 0.81$. This is illustrated by Figure 4.

For $q > 0.81$ only the zero fixed point survives. It means that 19% of non-voting persons are enough to make it impossible for the Green group to win. The unstable fixed point which can drive them to power has disappeared. The self-elimination is then much more stronger than for the 4-cell case.

We see that the abstention phenomenon will always produce a whole series of tied cases whatever the cell size, making the 4-cell findings more robust against larger sized cells.

This work is only a snapshot of reality, but yet it grasps some essential and surprising mechanisms of majority rule voting. It could shed some new light on the sudden collapse in 1989/90 of eastern European communist regimes which were based, at least in theory, on the principle of democratic centralism using small sized cells. Our dynamics may also provide a new view on management problems in big enterprises as well as on the question of the dynamics of innovation.

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